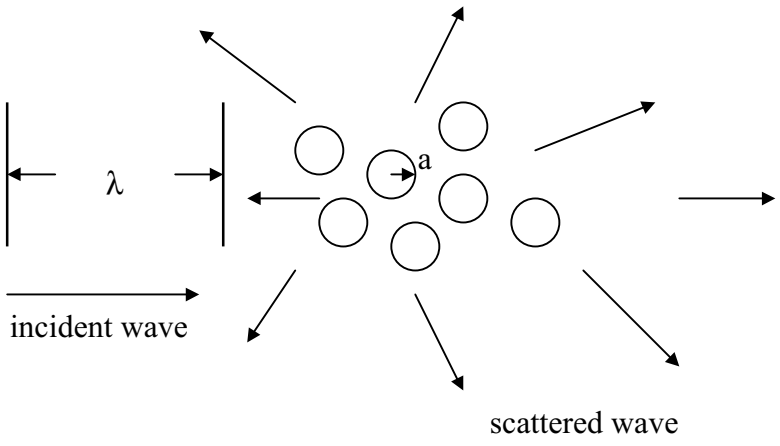
Chapter 3 : Scattering, Attenuation and Speckle

I. Scattering

- Pulse echo imaging relies on reflected waves (echoes) to form an image and the reflection occurs due to acoustic impedance differences. In the case that the objects which cause reflection are much larger than the wavelength of the incident waves, it is also called *specular* reflection. Using typical impedance values in the body, we find that the energy reflected is about -20 to -50 dB lower than the energy of the incident waves. In other words, not much acoustic energy is lost at the boundary of two different types of soft tissue.

- On a microscopic scale when the objects are much smaller than the wavelength, mechanical properties inherent in tissue also scatter sound waves and this is termed *Rayleigh* scattering. In pulse echo imaging, the re-radiated energy scattered in the backward direction, i.e., backscattering, is of primary interest. Note that although this type of volumetric signals are very weak (even weaker than the surface reflections), it is of great important because it contains the intrinsic properties of the micro-structure of tissue.



- Based on the ratio of the scatterer size to wavelength (λ), we can define the following three regions :

  - Optical : $ka>>1$, where $k=2\pi/\lambda$ (wave number) and a is the scatterer's radius.
  - Rayleigh : $ka<<1$.
  - Oscillatory : region in between.

- A few parameters can be defined to help us quantify scattering. (1). Scatter cross section (σ_s) is defined as the ratio of the total power scattered by the object to the incident energy. Similarly, backscatter cross section (σ_b) can be defined as the ratio of the total power backscattered by the object to the incident energy. (2). For a collection of scatterers, backscatter coefficient (ε), which is defined as the backscattering cross section per unit volume of scatterers, is usually used to represent the backscattering efficiency. In some cases, the backscatter coefficient is further normalized to *solid angle* with an additional unit of sr^{-1} (a sphere is equal to 4π steradians).

- In the Rayleigh scattering region, a relatively simple approach can be taken to obtain an analytical expression for the scatter cross section (σ_s) by using principle of superposition and the Born approximation (ignoring secondary scattering inside the objects). Under these conditions, we have

$$\sigma_s \propto k^4 a^6,$$

  where k is the wave number and a is the radius of the object.

- The ultimate goal of studying ultrasonic scattering in the body is to characterize tissue quantitatively. Due to the complexity of biological tissues, however, it is extremely difficult to use simple analytical expressions to describe scattering in the body. In addition, dependencies on animal species, tissue types and experimental techniques still sometimes produce inconsistent *in vitro* results. In fact, the fundamental scattering structures in most of the tissues are still unknown to this date.

- Ultrasonic scattering in biological tissues is primarily determined by size and acoustic properties of tissue structures. It is believed that composition of cells, blood vessels and ductal networks plays an important role in the ultrasonic scattering structure. Additionally, the size of tissue structure may also be a dominant factor. For example, it has been shown that the volume of red blood cells affects ultrasonic scattering in blood to a great extent. However, for most biological tissues, which are typically more complex than blood, the scattering mechanisms have not been fully understood.

- (Very) Roughly speaking, assuming the frequency of the incident waves is f, the backscatter coefficient is proportional to f^4 for blood, f^3 for myocardium, and

$f^{1.5-2.5}$ for other soft tissues. Note that the backscattering from blood is the closest to Rayleigh scattering. Also note that as the frequency increases, signals backscattered from blood also increase faster than those backscattered from other soft tissues.

- Some typical numbers for blood and heart tissue

Frequency (MHz)	ε(mm^{-1}) heart tissue	ε(mm^{-1}) blood
2.5	4.3×10^{-5}	0.5×10^{-6}
3.75	1.5×10^{-4}	2.6×10^{-6}
5.0	5.0×10^{-4}	8.2×10^{-6}

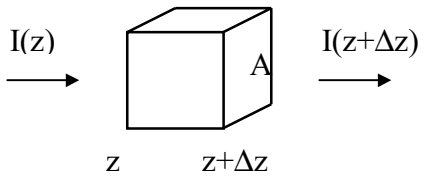
III. Attenuation

- Reflection and scattering result in signal loss as a sound wave propagates in tissue. However, this type of redistribution of energy only accounts for a small amount of attenuation. It is believed that the relaxation process, which causes absorption, is the major source of attenuation.

- Relaxation occurs when the pressure change resulting from a reduction of volume occupied is not quite in phase with the change in density, and energy is thereby changed to heat. In biological tissues, the product of ultrasonic absorption and wavelength is roughly constant in the diagnostic frequency range. In other words, it is reasonable to assume that the absorption is linearly proportional to the frequency.

- Fundamentally, the maximum penetration depth is determined by the frequency dependent attenuation and clinical safety requirements. Different clinical applications require different field of view (including penetration depth), thus requiring different ultrasonic carrier frequencies.

- Assuming *I(z)* is the incident wave and let the energy loss per unit length is proportional to the intensity, we have



$$A \cdot I(z+\Delta z) = A \cdot I(z) - 2\beta A \cdot I(z)\Delta z .$$

If $\Delta z \to 0$, then

$$-\frac{\partial I(z)}{\partial z} = 2 \cdot \beta I(z)$$
$$I(z) = I_0 e^{-2\beta z} .$$

Assuming attenuation is linearly proportional to frequency in tissue, we have

$$\beta = \alpha f .$$

Therefore, we can define the following characteristic propagation function for a plane wave propagating in tissue

$$H(z,f) = e^{-(\alpha f z + j2\pi f z / c)}$$

and we have the following relation for intensity

$$I(z,f) = I_0 \left| H(z,f) \right|^2 = I_0 e^{-2\alpha f z} .$$

- The attenuation in dB is defined as

$$-10\log_{10}\left(\frac{I(z,f)}{I_0}\right) = 20(\log_{10} e)\alpha f z = 8.69\alpha f z ,$$

where α is defined in (nepers)/cm/MHz. In dB, α_{dB} is in units of dB/cm/MHz and $\alpha_{dB} = 8.69\alpha_{nepers}$. In soft tissue, α_{dB} is typically assumed to be 0.5 dB/cm/MHz.

- In pulse echo imaging, signals reflected from a depth of *R* actually have a total propagation distance of *2R*. Therefore, assuming the center frequency of the sound waves is f_0, signals received from a depth of *R* is $\alpha_{dB} * 2R * f_0$ dB lower than the transmitted waves.

- Since attenuation is both frequency and depth dependent, the frequency contents of the transmitted sound waves are also affected as they propagate in tissue. Assume a wide band (as opposed to a continuous wave) Gausian pulse with the following power spectrum

$$\left|S_t(f)\right|^2 = e^{-(\frac{f-f_0}{\sigma})^2}.$$

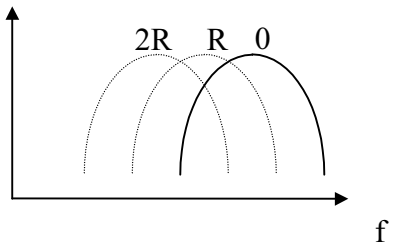
Also assuming specular and ignoring backscattering, the round trip received signal from a depth of *R* is

$$\left|S_r(R,f)\right|^2 = \left|S_t(f)\right|^2 e^{-4\alpha Rf} = e^{-(\frac{f-f_0}{\sigma})^2 - 4\alpha Rf}.$$

The above equation can be re-arranged as the following

$$\left|S_r(R,f)\right|^2 = e^{-(\frac{f-f_1}{\sigma})^2} e^{-4\alpha R(f_0-\sigma^2\alpha R)},$$

where $f_1 = f_0 - 2\sigma^2\alpha R$. In other words, the received signal is attenuated by the amount indicated in the second term and more importantly, the center frequency of the propagating pulse is shifted downward as the wave propagates. The bandwidth, on the other hand, remains the same. Therefore, the lateral resolution, which is a strong function of center frequency, is affected while the axial resolution, which is determined by the bandwidth, is not changed.



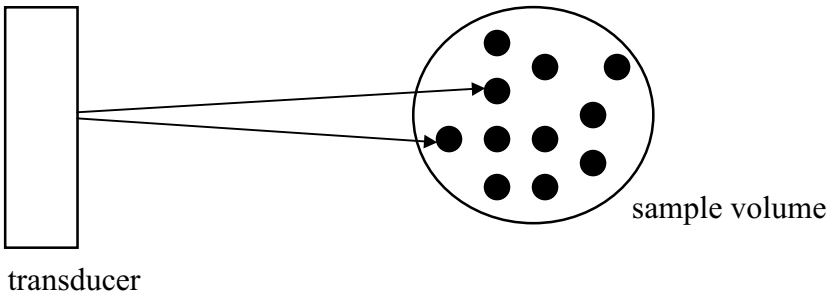
III. Speckle

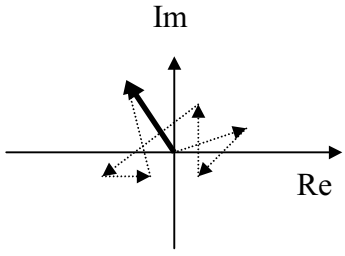
- Speckle is a common phenomenon in coherent (i.e., phase sensitive) imaging systems. It comes from coherent interference of scatterers (i.e., linear summation by the transducer of echoes from scatterers) and it appears as a granular structure superimposed on the image. Speckle is an artifact degrading target visibility and does not represent any inherent tissue properties. It is one of the primary limitations on detecting low contrast lesions in ultrasonic imaging.

- The statistical properties of speckle are based on the approach taken by J.W. Goodman for laser speckle. In ultrasound, speckle occurs when the size of

scatterers is small compared to a wavelength and there are many scatterers within one sample volume. This is true for diagnostic ultrasound since most scatterers in soft tissue have a size less than 100μm and the size of a typical sample volume (i.e., spatial resolution) is at the order of mm. In this case, the speckle pattern statistics are independent of the scattering structures and are a function only of the imaging system and its relative distance to the target.

- Speckle can be modeled by a random walk in the complex plane where each step in the walk represents the signal received by the transducer from a single scatterer within the resolution volume. Since these scatterers are in the same sample volume, the signals from them are coherently summed when received by the system. The sum can therefore be represented by a complex number shown in the following drawing. Note that the amplitude of each individual phasor represents the strength of the signal from the particular scatterer, whereas the phase of the phasor is related to the propagation delay (i.e., distance between the scatterer and the transducer).





- Let $a_k\ (1 \le k \le N)$ be the signal received from a particular scatterer with a phase θ_k, we have the following equations for the summed signal A:

$$\mathrm{Re}\{A\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \cos\theta_k$$
$$\mathrm{Im}\{A\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \sin\theta_k \quad .$$

Assuming the following statistical properties:

- The amplitude a_k and the phase θ_k of the *k*th elementary phasor are statistically independent of each other and of the amplitudes and phases of all other individual phasors.
- There are many scatteres in the sample volume and their locations are randomly distributed. Therefore, the phases θ_k are uniformly distributed between -π to π.

It then follows from the *central limit theorem* that as $N \to \infty$ (practically, $N \geq \sim 10$), *Re{A}* and *Im{A}* are asymptotically Gaussian. Combining this with other first and second order statistics, we find the joint probability density function of the real and imaginary parts of *A* is

$$p_{\mathrm{Re}\{A\}.\mathrm{Im}\{A\}} = \frac{1}{2\pi\sigma^2} e^{-\frac{\mathrm{Re}\{A\}^2+\mathrm{Im}\{A\}^2}{2\sigma^2}},$$

where

$$\sigma^2 = \frac{1}{N}\sum_{k=1}^{N}\frac{|a_k|^2}{2}.$$

This function is also known as a circular Gaussian density function.

- It is then straightforward to see that the intensity of the summed signal

  $I \equiv \mathrm{Re}\{A\}^2 + \mathrm{Im}\{A\}^2$ is exponentially distributed, i.e., (for $I \geq 0$)

$$p_I = \frac{1}{2\sigma^2} e^{-\frac{I}{2\sigma^2}}$$

  and the amplitude $E \equiv \sqrt{I}$ is Rayleigh distributed, i.e., (for $I \geq 0$)

$$p_E = \frac{E}{\sigma^2} e^{-\frac{E^2}{2\sigma^2}}.$$

- Some interesting properties for the above two density functions are listed as follows:

$$SNR_I \equiv \frac{\rangle I \langle}{\sigma_I} = 1$$
$$SNR_E \equiv \frac{\rangle E \langle}{\sigma_E} = \frac{\left(\pi\sigma^2/2\right)^{1/2}}{\left((4-\pi)\sigma^2/2\right)^{1/2}} \approx 1.91 \quad ,$$

where σ_I and σ_E are standard deviations of the intensity and the amplitude, respectively.

Therefore, speckle noise can be viewed as a multiplicative noise, where the noise increases as the mean increases. In other words, stronger signals also suffer from higher noise and hence they are not easier to be detected.

- Ultrasonic images are often displayed on a logarithmic scale so that signals across a wide dynamic range can be shown at the same time. Furthermore, the speckle noise also becomes *additive* on a logarithmic scale. Define *D* as

$$D(dB) = f(I) \equiv 10\log_{10}(\frac{I}{I_0}),$$

where I_0 is an arbitrary reference signal. Expanding the function *f* in a Taylor series about the statistical mean $\langle I \rangle$, we have

$$D = f(\langle I \rangle) + \left(I - \langle I \rangle\right) f'\left(\langle I \rangle\right) + R,$$

where *R* is a remainder. Ignoring *R*, we have

$$\sigma_D{}^2 \approx f'(\langle I \rangle)^2 \sigma_I{}^2 = \left(\frac{10}{\ln 10}\right)^2 \frac{\sigma_I{}^2}{\langle I \rangle^2}.$$
$$\sigma_D \approx 4.34(dB)$$

In other words, speckle noise in a logarithmically processed display becomes a fixed additive noise. This noise (4.34dB) fundamentally limits the detectability of low contrast lesions using diagnostic ultrasound.

- There are numerous techniques developed to reduce the speckle noise. Almost all of them, however, result in degradation in spatial resolution. More details will be covered in Chapter 7.